**2019 Nobel Prize Posthumously Awarded to E. Ising, W. Lenz and L. Onsanger**

*"For their contributions to statistical physics, namely their work on the model now named after Ising with its diverse applications including a description of ferromagnetism, universality and as a testing bed for the third law of thermodynamics."*

*by Ethan van Woerkom*

The Nobel committee has, for the first time in its history, posthumously awarded the Nobel Prize to the German physicists Ernst Ising, Wilhelm Lenz and Norwegian-American physicist Lars Onsanger. This exception is presumably justified by the pervasiveness of this model throughout statistical physics and other fields, ranging from ferromagnets, to ecology.

**"Umklappende Stabmagnet"-"Flipping Bar Magnet"**

The history of the Ising Model goes back to Lenz's initial formulation in his 1920 paper [1]. In this work he remarks that low temperature measurements from Kamerlingh Onnes confirm Curie's Law $M = \frac{C \cdot B}{T}$ for many paramagnetic salts, and that Pierre Weiss's explanations of the Curie Law were severally unsatisfactory for these materials. One explanation assumes that they can be treated the same as paramagnetic gases, with freely rotating magnetic moments, in the second he assumes small vibrations about equilibrium. The first explanation is flawed since it has become clear that atoms are not free to rotate in salts, and the second explanation yields too small an effect.

Lenz, inspired by the spontaneous and directionally quantised magnetisation directions of magnetite and pyrrhotite, proposes a simple model: he treats each atom as an elementary bar magnet with fixed magnetic moment μ and then proceeds to examine a single such bar magnet, which constrained by large potential forces, can only point in a single direction, either positively, or negatively. Thermal fluctuations can cause these magnets to flip randomly. One would therefore expect the net magnetic moment J of a crystal to be zero. Applying a magnetic field H however, will split the energy levels of positive and negative direction, and by applying the Boltzmann distribution, he arrives at the result $\bar{\mu} = \mu \cdot \tanh a\,,\ a = \frac{\mu H}{kT}$, which approximates at high temperatures to $\bar{\mu} = \frac{\mu^2 H}{kT}$, which confirms the Curie Law for paramagnetism.

Finally, Lenz speculates that in ferromagnetic bodies, the energy levels of a magnet may depend on the orientation of the neighbours, and in this way "emerges a natural direction pertaining to the crystal structure and hence spontaneous magnetisation". It is in these last few sentences of his paper that the groundwork of and necessity for the exploration of the Ising model is laid.

**The 1-Dimensional Ising Model**

Ising, as a PhD student of Lenz's, was then given this problem, and solved it for the 1D case, for a chain of 'elementary magnets', under the simplifying assumption that only neighbours interact, in his 1925 paper [2,3]. Here, he assumed that aligning the magnetic moments of neighbours decrease the energy, and anti-aligned neighbours increase energy.

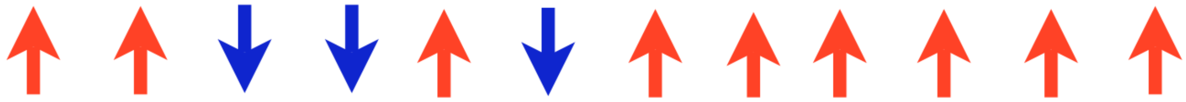

**Fig. 1 Example 1-D Ising Model [6]**

The hope was that, as in real life, a phase transition could be found at a critical temperature, below which the material is ferromagnetic, and above which the material is paramagnetic.

To their disappointment [3], no such transition was found. In the result for the total magnetic moment of the system, there was again none when no external field is applied, and the system was paramagnetic throughout for all T. A very clumsy approximation to the three-dimensional case yielded exactly the same result. It was suspected by those including Wolfgang Pauli however [3], that a fully-fledged higher-dimensional model would display a phase change.

**Attack on the 2-Dimensional Problem**

The original problem was proposed (1920) and solved (1925), during the very earliest time of quantum mechanics, when concepts of “quantisation” had already become very important, but the full theoretical machinery and understanding had not yet developed. Solving the problem in the context of the ‘old’ quantum mechanics therefore did not happen, and only after many extensive developments of the ‘new’ quantum mechanics, and important contributions by Kramers and Wannier [3], even finding the critical temperature, did Onsanger manage to completely solve the problem in a full 33-page offensive[4] using the ‘transfer-matrix’ method in 1944. He managed to find the partition function and so the specific heat. This 2D Ising model phase transition turns out to constitutes a universality class [3]. Pauli apparently called this the only interesting development in theoretical physics during WW2 in a conversation between with Hendrik Casimir [3]!

**Connections to Third Law of Thermodynamics**

The aligning of spins at low temperature, bringing entropy to zero, is an excellent demonstration of the third law of thermodynamics. Furthermore, certain setups of the Ising Model beyond the ‘elementary’ case, can demonstrate an exception to the third law of thermodynamics. Non-zero entropy at zero temperature is realisable within the model. This excess entropy was first demonstrated by Wannier in 1950 [5], using a triangular lattice structure.

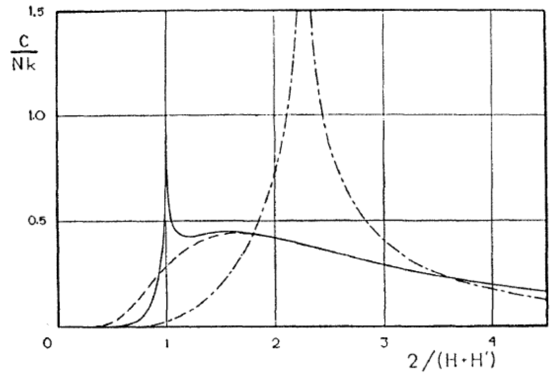


FIG. 7. Specific heats for varying degrees of anisotropy. ——— $J'/J=1/100$; — — — — — $J'/J=1$ (quadratic crystal); - - - - - - $J'=0$ (linear chain).

**Fig. 2 Specific heat diagram taken from Onsanger (1944) demonstrating universal phase transitions**

**Conclusion**

The Ising Model went through different stages, with the first important conceptual formulation by Lenz used to explain paramagnetism. Ising himself subsequently mathematically described the model and solved it for the 1D case, highlighting its deficiencies. Kramers, Wannier, and Onsanger together managed to bring it to its full fruition as a tool for the description of ferromagnetism in the context of ‘modern’ quantum theory. These facts together with the rich field of theoretical physics that sprang forth from this model, show that these three men rightly deserve the prize.

**References**

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